Plasma turbulence simulations of the tokamak scrape-off layer

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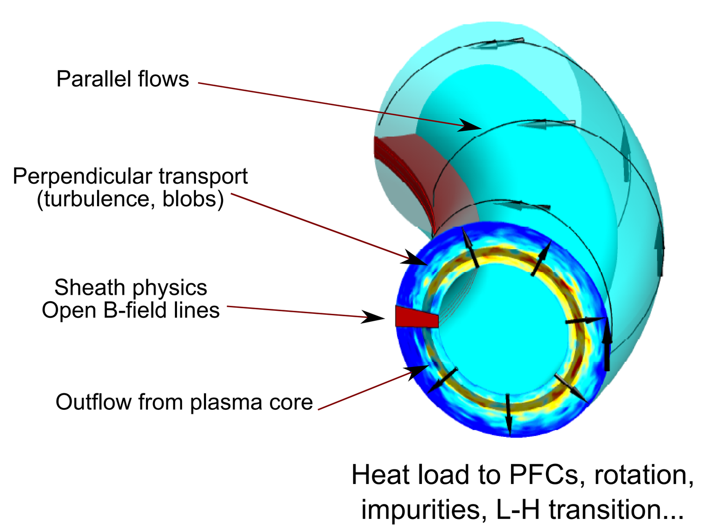
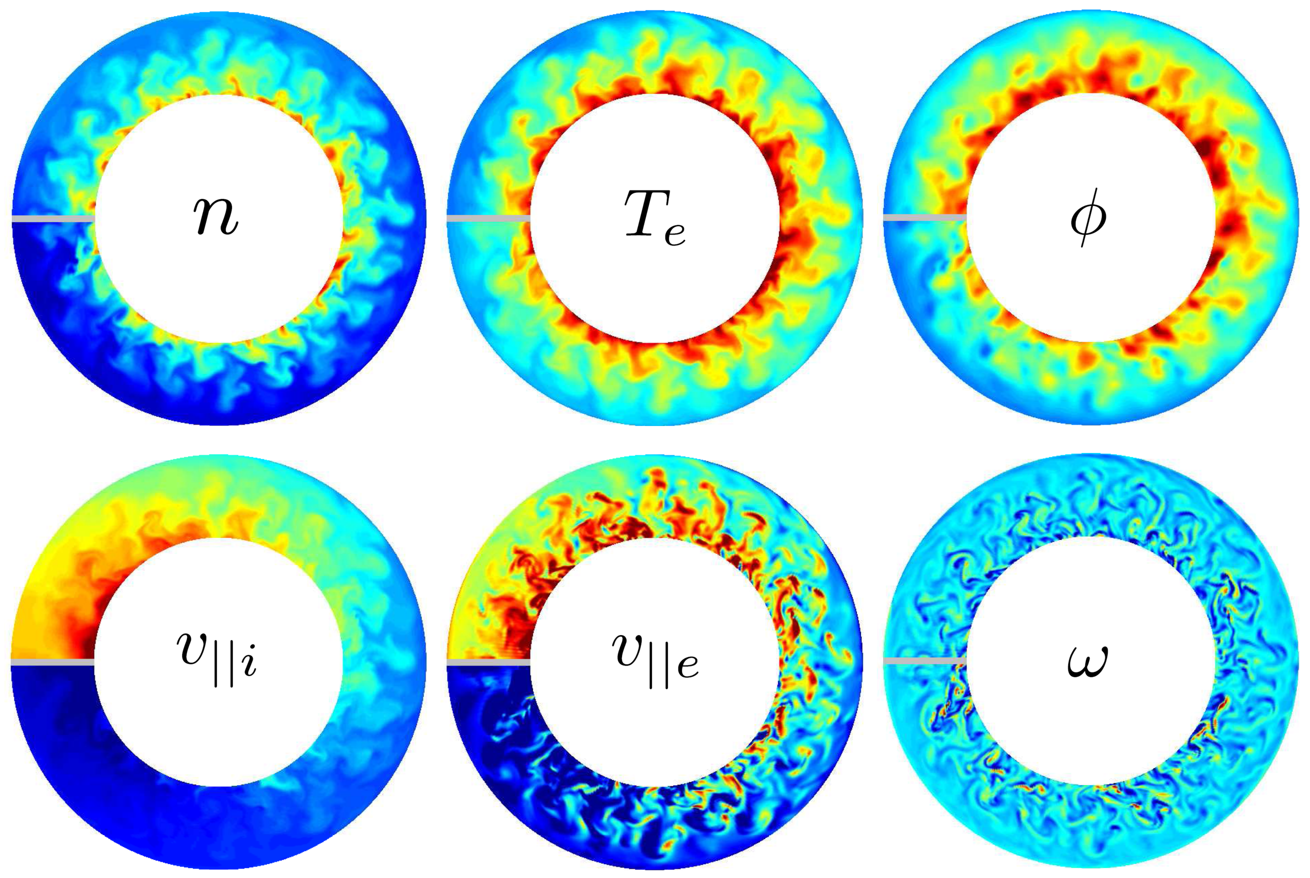
Introduction



- In the tokamak scrape-off layer (SOL) magnetic field lines are open, channeling heat onto device wall
- Large computational effort devoted to understand width of the heat flux channel using Global Braginskii Solver (GBS) plasma turbulence code
- SOL dynamics studied via direct numerical simulations capable of resolving turbulent dynamics at experimental parameters
- Latest numerical developments in GBS
  - Fast, efficient discretization of parallel dynamics
  - Matrix-free parallel multigrid solver for the Poisson equation
- Using new numerical approach, simulations of even larger tokamaks are possible using $\sim 10^4$ CPUs

GBS, a tool to study SOL turbulent plasma dynamics

- Flux-driven plasma turbulence code to study SOL heat and particle transport
- Code fully verified using method of manufactured solutions [Riva *et al.*, PoP (2014)]
- Spatial discretization using finite differences, RK4 integration in time
- Parallelized using domain decomposition in x and z axes
- For medium tokamak size $\rho_\star^{-1} \approx 2000$, excellent parallel scalability up to 1024 cores



Drift-reduced fluid equations for plasma turbulence

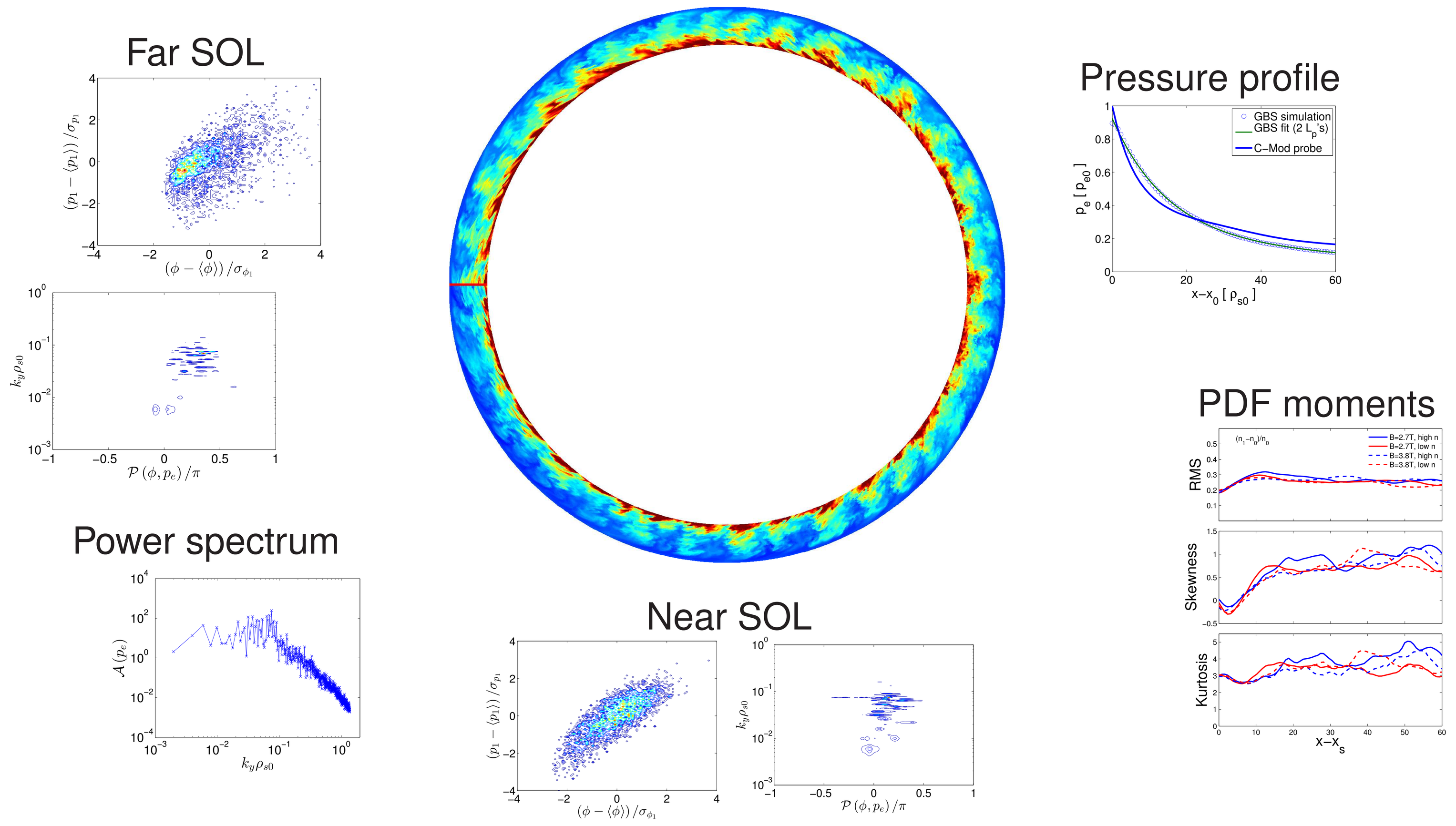
- Low-frequency, collisional, electromagnetic turbulence driven by plasma gradients
- Large fluctuations $\mathcal{O}(1)$, no length scale separation
- Drift-reduced Braginskii eqns with orderings $k_\perp \gg k_\|$, $d/dt \ll \omega_{ci}$ [Ricci *et al.*, PPCF 2012]:

$$
\begin{aligned}
\frac{\partial n}{\partial t} &= -\rho_\star^{-1}[\phi, n] + \frac{2}{B}\left[C(p_e) - nC(\phi)\right] - \nabla_\| \cdot (n v_{\|e}) + S_n \\
\frac{\partial \nabla_\perp^2 \phi}{\partial t} &= -\rho_\star^{-1}[\phi, \nabla_\perp^2\phi] - v_{\|i}\nabla_\|\nabla_\perp^2\phi + \frac{B^2}{n}\nabla_\| j_\| + \frac{2B}{n}C(p) \\
\frac{\partial \chi}{\partial t} &= -\rho_\star^{-1}[\phi, v_{\|e}] - v_{\|e}\nabla_\| v_{\|e} + \frac{m_i}{m_e}\left(\nu\frac{j_\|}{n} + \nabla_\|\phi - \frac{1}{n}\nabla_\| p_e - 0.71\nabla_\| T_e\right) \\
\frac{\partial v_{\|i}}{\partial t} &= -\rho_\star^{-1}[\phi, v_{\|i}] - v_{\|i}\nabla_\| v_{\|i} - \frac{1}{n}\nabla_\| p \\
\frac{\partial T_e}{\partial t} &= -\rho_\star^{-1}[\phi, T_e] - v_{\|e}\nabla_\| T_e + \frac{4}{3}\frac{T_e}{B}\left[\frac{1}{n}C(p_e) + \frac{5}{2}C(T_e) - C(\phi)\right] + \frac{2}{3}T_e\left[0.71\nabla_\| j_\| - \nabla_\| v_{\|e}\right] + S_{T_e} \\
\frac{\partial T_i}{\partial t} &= -\rho_\star^{-1}[\phi, T_i] - v_{\|i}\nabla_\| T_i + \frac{4}{3}\frac{T_i}{B}\left[C(T_e) + \frac{T_e}{n}C(n) - C(\phi)\right] + \frac{2}{3}T_i\left(v_{\|i} - v_{\|e}\right)\frac{\nabla_\| n}{n} - \frac{2}{3}T_i\nabla_\| v_{\|e} - \frac{10}{3}\frac{T_i}{B}C(T_i) \\
\nabla_\perp^2\psi &= n(v_{\|i} - v_{\|e}) = j_\|,\ \chi = v_{\|e} + \frac{m_i}{m_e}\frac{\beta}{2}\psi,\ \rho_\star = \rho_s/R \\
\nabla_\| f &= \mathbf{b}_0 \cdot \nabla f + \rho_\star^{-1}\frac{\beta}{2}[\psi, f]
\end{aligned}
$$

- Normalized units used throughout: $L_\perp \to \rho_s$, $L_\| \to R$, $t \to R/c_s$, $\nu = ne^2 c_s/(m_i \sigma_\| R)$

SOL turbulent dynamics revealed through computer simulations

- GBS simulations at realistic parameters reproduce features found in SOL of tokamak discharges
  - Pressure profile is a decaying exponential with 2 decay lengths
  - Large fluctuations $\sim 30\%$, skewed PDF shows presence of blobs
  - Two distinct regions: drift dominated vs. interchange dominated
- Detailed comparison between code and experimental measurements in progress
  - Dedicated experiments carried out at Alcator C-Mod (MIT) using state of the art diagnostics



- Analysis of a large simulation scan revealed turbulent saturation mechanism, non-linear instability regimes, equilibrium electric field, effects of parallel dynamics...
- SOL width can be calculated analytically by considering *gradient removal saturated turbulence*:

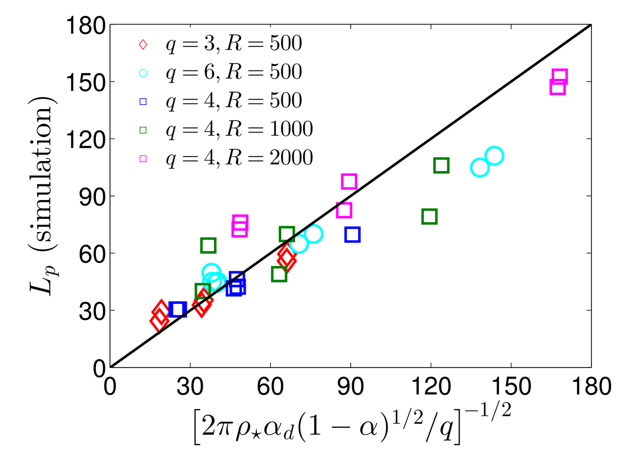
$$L_p = q\frac{\gamma}{k_\theta}, \qquad \gamma_b = \sqrt{2/(\rho_\star L_p)}, \qquad k_b = \sqrt{(1-\alpha)/(\nu\gamma_b)}/q$$

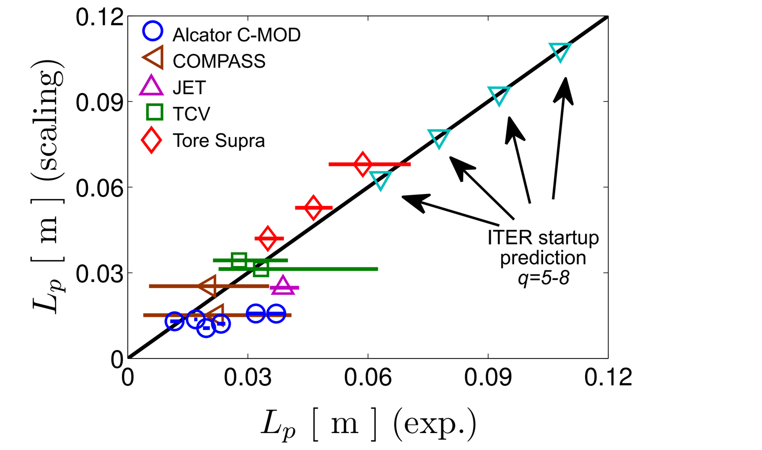
$$\alpha_d^{-1} = 2^{7/4}\nu^{1/2}(\rho_\star L_p)^{1/4}\pi q) \qquad \alpha = \frac{q^2\beta}{\rho_\star L_p}$$

- Dimensionless and engineering parameter scalings of the SOL width follow [Halpern *et al.*, NF 2013]:

$$\boxed{L_p = \left[2\pi\rho_\star\alpha_d(1-\alpha)^{1/2}/q\right]^{-1/2}}$$

$$\boxed{L_p \approx 7.2 \times 10^{-8} q^{8/7} R^{5/7} B_\phi^{-4/7} T_{e0}^{-2/7} n_{e0}^{2/7}(1 + T_i/T_e)^{1/7}\ [\,\mathrm{m}\,]}$$





- Scalings obtained from least-squares-fitting of all simulation data verify our theory:

$$L_p = 0.42\, q^{0.55}\rho_\star^{-0.53}\alpha_d^{-0.32}(1-\alpha)^{-0.24} \to L_p \sim q^{0.98}R^{0.63}B^{-0.56}[\,\mathrm{m}\,]$$

Recent numerical developments

Parallel dynamics discretization schemes

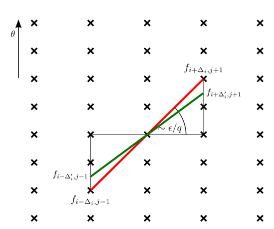
Description of parallel dynamics essential for efficient and stable plasma turbulence codes

- Turbulence strongly anisotropic, elongated turbulent structures aligned to *B*-field
- Basic wave phenomena regulating plasma dynamics $\to$ shear-Alfvén waves
- We test field-aligned and non-field aligned schemes to find optimal algorithm for GBS

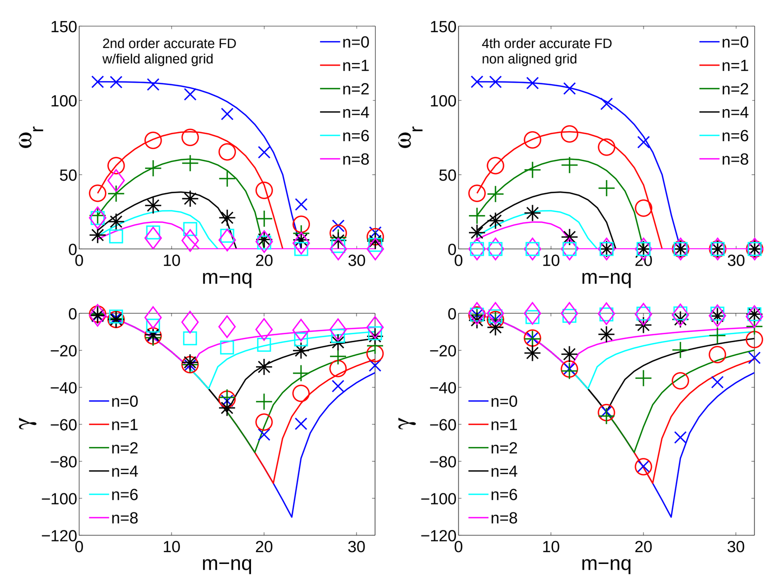
- Study parallel dynamics using simplified model

$$\frac{\partial \nabla_\perp^2\phi}{\partial t} = -\nabla_\| v_{\|,e}\ , \qquad \frac{\partial v_{\|,e}}{\partial t} = \frac{m_i}{m_e} + \frac{4}{3}\eta\nabla_\|\phi$$

- Simple analytical solution $\omega^2 = \omega_0^2 - \gamma^2$, $\omega_0 = \sqrt{m_i/m_e}k_\|/k_\perp$, $\gamma = 2\eta k_\|^2/3$
- Evolve perturbations $\sim \sin(my - nz)$ using different numerical schemes



- Evaluate numerical ω, γ using simple code:



Field aligned and 4th order FD schemes reproduce parallel dynamics accurately and efficiently

Parallel, matrix-free multigrid solver

Efficient, scalable inversion of time dependent operators, *e.g.*:

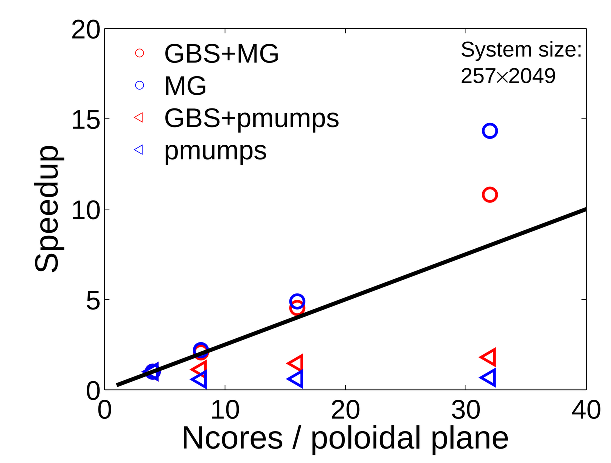
- Electromagnetic Ohm's law

$$\left[\nabla_\perp^2 - \frac{\beta}{2}\frac{m_i}{m_e}n\right]v_{\|e} = S_{v_{\|e}}$$

- Non-Boussinesq polarization equation

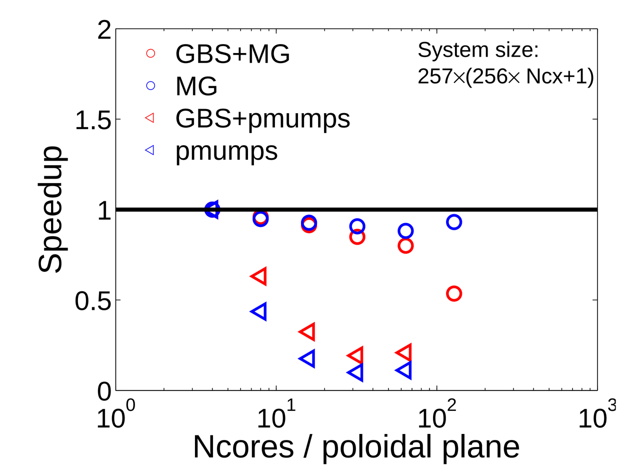
$$\left[\nabla_\perp^2 + \nabla_\perp \ln n \cdot \nabla_\perp\right]\phi = S_\phi$$

- Avoid LU decomposition in every time step



Developed multigrid solver in 2D Cartesian grid mapped to 2D domain decomposition

- FD matrix partitioned using stencil notation
- Full weighting restriction, bilinear interpolation for prolongation, damped Jacobi relaxation, $\Omega = 0.9$
- Carry out simple test with $\nabla_\perp^2\phi = \omega$, weak and strong scalings in MonteRosa shown below



Conclusions

- Numerical studies of tokamak turbulence improved understanding of plasma-wall interaction
- Simulations show many features found in experimental measurements
- Developed predictive theory for SOL width of inner-wall limited tokamak plasmas
- Analyzed wave dispersion and damping with numerical schemes appropriate for X-point geometry
- Demonstrated super-linear GBS speedup using parallel multigrid solver

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Outlook and GBS development plans

- Validate SOL model against dedicated experiments in Alcator C-Mod
- Parallelize GBS in y axis $\to$ factor 10 speedup
- Use several numerical approaches to model magnetic separatrix / X-point
- Port GBS to OpenMP/MPI hybrid, then to GPU architecture
- Carry out kinetic simulations of plasma biasing $\to$ turbulence control